Retrieving squeezing from classically noisy light in second-harmonic generation

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We report the results of a study of the quantum noise properties of a squeezing system involving a three-level laser pumping two similar second-harmonic-generating crystals. We show that squeezing that has been obscured by intensity and phase noise from the pump laser may be retrieved by difference detection of both second-harmonic outputs. Similarly, the squeezed vacuum formed by combining the two outputs on a 50/50 beam splitter will be squeezed at frequencies that are classically noisy in the individual beams.

1. INTRODUCTION

Often the squeezing produced by an optical system is limited or destroyed by the presence of classical noise sources. For example, the presence of intensity and phase noise in the pump laser is known to destroy the squeezing from second-harmonic generation.[1] Usually one can avoid this problem by observing the squeezing in a region of the spectrum where the pump laser is shot-noise limited. However, for a number of applications squeezing at frequencies of a few tens of kilohertz is desirable. Lasers are usually noisy at such frequencies.

It has been suggested that if two independently squeezed beams are produced with common classical noise then it is possible to subtract the classical noise while retaining the squeezing.[2] This result has been used to explain the surprisingly good squeezing observed at low frequencies in the fiber ring experiment of Bergman and Haus.[3]

Consider a system that emits two optical beams with independent quantum noise but correlated classical noise, as represented in Fig. 1. We assume that the mean photon number and the variance of the quantum noise are numerically the same for both beams, i.e., $n = \langle \hat{u}^\dagger \hat{u} \rangle = \langle \hat{v}^\dagger \hat{v} \rangle$ and $\Delta n^2 = \langle \hat{u}^\dagger \hat{u} \hat{u}^\dagger \hat{u} \rangle - \langle \hat{u}^\dagger \hat{u} \rangle^2 = \langle \hat{v}^\dagger \hat{v} \hat{v}^\dagger \hat{v} \rangle - \langle \hat{v}^\dagger \hat{v} \rangle^2$, where $\hat{u}$ and $\hat{v}$ are the annihilation operators for the two beams. Correlated classical noise in the quantum states is modeled by multiplying both operators by the classical stochastic function $1 + \delta(t)$. We assume that the classical noise is a small perturbation $[1 \gg \delta(t)]$ with zero mean ($\langle \delta \rangle = 0$) and variance $\Delta \delta^2 = \langle \delta^2 \rangle$.

The beams are detected by two photodetectors, and their respective photocurrents are electronically added and subtracted. We get the following results for the variances of the sum $[n_+ = (1 + \delta)(\hat{u}^\dagger \hat{u} + \hat{v}^\dagger \hat{v})]$ and difference $[n_- = (1 + \delta)(\hat{u}^\dagger \hat{u} - \hat{v}^\dagger \hat{v})]$ photocurrents:

$$\Delta {n_+}^2 = 2(1 + \Delta \delta^2)\Delta n^2 + \Delta \delta^2 (2n)^2 \cong 2\Delta n^2 + \Delta \delta^2 (2n)^2 , \tag{1a}$$

$$\Delta {n_-}^2 = 2(1 + \Delta \delta^2)\Delta n^2 \cong 2\Delta n^2 . \tag{1b}$$

The standard quantum limit is the noise level for which the variance equals the mean. Hence squeezing will be detected if $\Delta {n_\pm}^2 < 2n$. Suppose that the quantum noise of the two beams is squeezed ($\Delta n^2 < n$). The sum photocurrent gives the total noise of the two beams and will not show squeezing if the classical noise term [second term in relation (1a)] is too large. However, the variance of the difference current [relation (1b)] is independent of the classical noise. Hence difference detection of the two beams will retrieve any underlying squeezing in spite of the fact that neither of the beams is squeezed individually. Similarly one can show that if the two beams are combined on a 50/50 beam splitter the null port will produce a squeezed vacuum. Note that it is essential that the quantum noise of the two beams be independent. For example, if the two beams are produced by beam splitting a single beam, they will be correlated. Difference detection will then remove both the classical noise and the quantum noise, leaving only vacuum noise at the standard quantum limit.

It is not clear from this simple argument under what specific conditions beams with the required properties would be produced. In the following we examine the experimentally realistic situation of two second-harmonic-generating crystals pumped by a single laser. We show, using a fully quantum-mechanical model, that the beams produced by this system have the correct properties such that squeezing can indeed be retrieved by use of this system. The paper is arranged in the following way: In Section 2 we set up the model and obtain solutions. In Section 3 we present our results. In Section 4, as a counterexample, we briefly discuss an apparently similar system from which squeezing cannot be retrieved. A summary and a discussion of the experimental implications are presented in Section 5.

2. MODEL AND SOLUTION

Figure 2(a) is a schematic representation of the experimental setup analyzed. The atomic level scheme of the active laser atoms is depicted in Fig. 2(b). Our laser model consists of N three-level atoms interacting with an optical ring cavity mode through the resonant Jaynes–Cummings Hamiltonian

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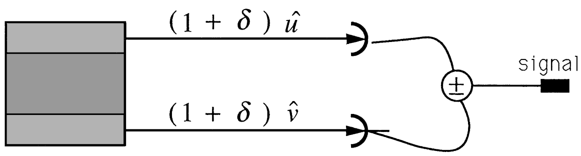


Fig. 1. Schematic representation of a system that emits beams with independent quantum noise and correlated classical noise.

$$\hat{H}_l = i\hbar g(\hat{a}^\dagger \hat{J}_{23}{}^- - \hat{a}\hat{J}_{23}{}^+), \tag{2}$$

where carets indicate operators, g is the dipole coupling strength between the atoms and the cavity, $\hat{a}$ and $\hat{a}^\dagger$ are the laser cavity mode annihilation and creation operators, and $\hat{J}_{ij}{}^-$ and $\hat{J}_{ij}{}^+$ are the collective Hermitian conjugate lowering and raising operators between the $|i\rangle$th and $|j\rangle$th levels of the lasing atoms. The field phase factors have been absorbed into the definition of the atomic operators.

The two second-harmonic generating (SHG) cavities will be referred to as SHG crystal 1 and SHG crystal 2. The fundamental cavity mode operators of the kth SHG cavity, $\hat{b}_k{}^\dagger$ and $\hat{b}_k$, interact with the second-harmonic cavity mode operators, $\hat{c}_k{}^\dagger$ and $\hat{c}_k$, through the Hamiltonian[4]

$$\hat{H}_{sk} = i\hbar \frac{\epsilon_k}{2} (\hat{b}_k{}^{\dagger 2}\hat{c}_k - \hat{b}_k{}^2\hat{c}_k{}^\dagger), \tag{3}$$

where ϵ_k is the coupling constant for the interaction between the two modes through the nonlinear crystal. Detuning between the laser mode and the kth fundamental SHG mode is $\Delta\omega_k$.

Following standard techniques,[5,6] we couple the lasing atoms and cavities to reservoirs and derive a master equation for the reduced density operator $\hat{\rho}$ of the system. We model the driving of the SHG cavities by the laser, using the cascaded quantum system formalism of Carmichael[7] and Gardiner.[8] Included in the laser model are atomic spontaneous emission from level $|3\rangle$ to level $|2\rangle$ and from level $|2\rangle$ to level $|1\rangle$, at rates γ_{23} and γ_{12}, respectively. Incoherent pumping occurs at a rate Γ. γ_p is the rate of collisional- or lattice-induced phase decay of the lasing coherence. The laser cavity damping rate that is due to the output port that pumps the kth SHG cavity is $2\kappa_{ak}$. The cavity decay rate for the fundamental mode of the kth SHG cavity is $2\kappa_{bk}$. This is also the port through which pumping of the fundamental occurs. The cavity decay rate for the second-harmonic mode of the kth SHG cavity is $2\kappa_{ck}$. The resulting interaction picture master equation is

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = {}& \frac{1}{i\hbar}[\hat{H}_l, \hat{\rho}] + \frac{1}{i\hbar}[\hat{H}_{s1}, \hat{\rho}] + \frac{1}{i\hbar}[\hat{H}_{s2}, \hat{\rho}] \\
&+ \frac{1}{2}(\gamma_{12}L_{12} + \gamma_{23}L_{23})\hat{\rho} + \frac{\Gamma}{2}(L_{13}\rho)^\dagger \\
&+ \frac{1}{4}\gamma_p[2(\hat{J}_3 - \hat{J}_2)\hat{\rho}(\hat{J}_3 - \hat{J}_2) \\
&- (\hat{J}_3 - \hat{J}_2)^2\hat{\rho} - \hat{\rho}(\hat{J}_3 - \hat{J}_2)^2] \\
&+ (\kappa_{a1} + \kappa_{a2})(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) \\
&+ \sum_{k=1,2}[\kappa_{bk}(2\hat{b}_k\hat{\rho}\hat{b}_k{}^\dagger - \hat{b}_k{}^\dagger\hat{b}_k\hat{\rho} - \hat{\rho}\hat{b}_k{}^\dagger\hat{b}_k) \\
&+ \kappa_{ck}(2\hat{c}_k\hat{\rho}\hat{c}_k{}^\dagger - \hat{c}_k{}^\dagger\hat{c}_k\hat{\rho} - \hat{\rho}\hat{c}_k{}^\dagger\hat{c}_k) \\
&+ 2\sqrt{\kappa_{ak}\kappa_{bk}}(\hat{a}\hat{\rho}\hat{b}_k{}^\dagger + \hat{b}_k\hat{\rho}\hat{a}^\dagger - \hat{\rho}\hat{a}^\dagger\hat{b}_k - \hat{b}_k{}^\dagger\hat{a}\hat{\rho})], \\
L_{ij}\hat{\rho} = {}& (2\hat{J}_{ij}{}^-\hat{\rho}\hat{J}_{ij}{}^+ - \hat{J}_{ij}{}^+\hat{J}_{ij}{}^-\hat{\rho} - \hat{\rho}\hat{J}_{ij}{}^+\hat{J}_{ij}{}^-) .
\end{aligned} \tag{4}$$

We may obtain the semiclassical equations of motion directly from the master equation by making the approximation of factorizing expectation values. They are

$$\begin{aligned}
&\dot{\tilde{\alpha}} = \tilde{g}_{23}\tilde{J}_{23} - (\kappa_{a1} + \kappa_{a2})\tilde{\alpha}, \qquad \dot{\tilde{c}}_i = -\kappa_{ci}\tilde{c}_i - \frac{\epsilon_i}{2}\tilde{\beta}_i 2, \\
&\dot{\tilde{\beta}}_i = \epsilon_i\tilde{\beta}_i{}^\dagger\tilde{c}_i - (\kappa_{bi} + i\Delta\omega)\tilde{\beta}_i - 2\sqrt{\kappa_{ai}\kappa_{bi}}\,\tilde{\alpha}, \\
&\dot{\tilde{J}}_{23} = \tilde{g}_{23}(\tilde{J}_3 - \tilde{J}_2)\tilde{\alpha} - 1/2(\gamma_{23} + \gamma_{12} + 2\gamma_{p1})\tilde{J}_{23}, \\
&\dot{\tilde{J}}_2 = \tilde{g}_{23}(\tilde{J}_{23}\tilde{\alpha}^\dagger + \tilde{J}_{23}{}^+\tilde{\alpha}) + \gamma_{23}\tilde{J}_3 - \gamma_{12}\tilde{J}_2, \\
&\dot{\tilde{J}}_3 = -\tilde{g}_{23}(\tilde{J}_{23}\tilde{\alpha}^\dagger + \tilde{J}_{23}{}^+\tilde{\alpha}) + \Gamma\tilde{J}_1 - \gamma_{23}\tilde{J}_3
\end{aligned} \tag{5}$$

and their conjugate equations. The absence of circumflexes indicates semiclassical quantities, and tildes indicate the following scaling with number of lasing atoms:

$$\begin{aligned}
&\alpha = \tilde{\alpha}N^{1/2}, \qquad J_{ij} = \tilde{J}_{ij}N, \qquad J_i = \tilde{J}_iN, \\
&g = \tilde{g}N^{-1/2}, \\
&\beta_i = \tilde{\beta}_iN^{1/2}, \qquad c_i = \tilde{c}_iN^{1/2}, \qquad \epsilon_i = \tilde{\epsilon}_iN^{-1/2}.
\end{aligned} \tag{6}$$

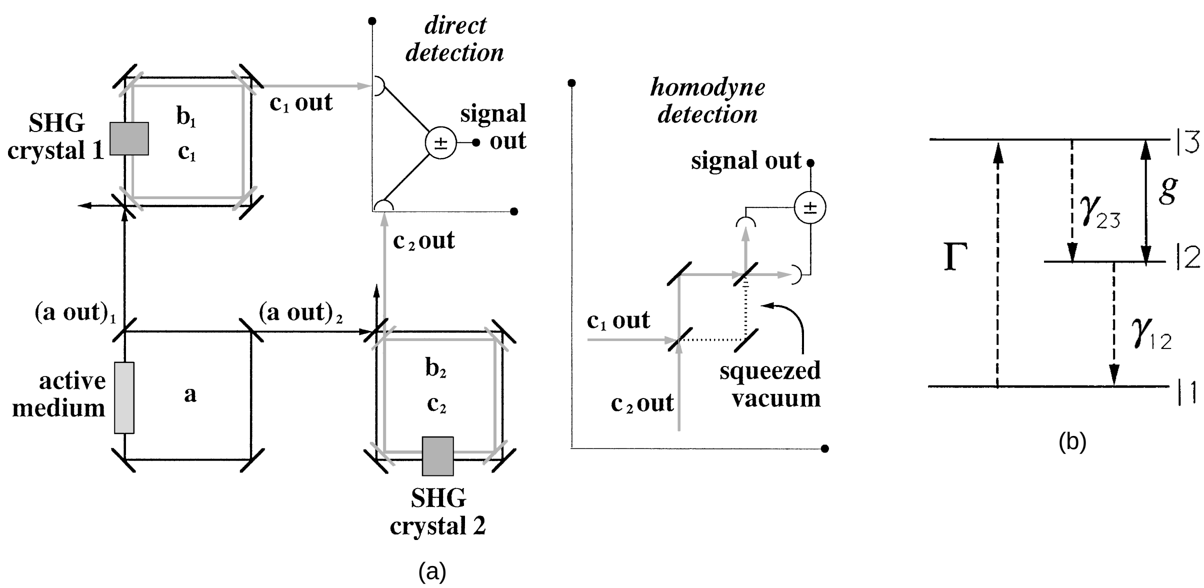


Fig. 2. (a) Schematic representation of the system analyzed, showing two detection methods. The spectra obtained by the two methods are equivalent. (b) Energy-level scheme of the laser atoms.

Setting the derivatives in Eqs. (5) to zero, we can solve for the semiclassical steady state. To solve for the squeezing spectrum we assume that the quantum fluctuations are sufficiently small that we can treat them as linear perturbations around the stable semiclassical steady state. This is appropriate for the levels of squeezing that we obtain. We write the full solutions in the form

$$\tilde{\alpha} = \tilde{\alpha}_0 + \delta\tilde{\alpha}\,, \qquad \tilde{J}_{ij} = \tilde{J}_{ij;0} + \delta\tilde{J}_{ij}\,, \qquad \tilde{J}_i = \tilde{J}_{i;0} + \delta\tilde{J}_i\,,$$
$$\tilde{\beta}_i = \tilde{\beta}_{0i} + \delta\tilde{\beta}_i\,, \qquad \tilde{c}_i = \tilde{c}_{0i} + \delta\tilde{c}_i\,, \tag{7}$$

where the subscript 0 indicates the steady-state solution to Eqs. (5). Note that the phases of the complex solutions for the laser's amplitude and coherence are undetermined; hence without loss of generality we take them to be real. Fixing this phase limits the time for which the approximation of small quantum fluctuations is valid. This is because the laser phase diffuses away from its initial phase. However, this should not limit the validity of the intensity statistics that are of interest here.

The spectral matrix $\mathbf{S}(\omega)$ is defined as the Fourier-transformed matrix of two times correlation functions of the small quantum perturbations ($\delta\tilde{\alpha}_j$), i.e.,

$$S_{ij} = \int_{-\infty}^{\infty} \exp(i\omega\tau)\langle\delta\tilde{\alpha}_i(t+\tau),\, \delta\tilde{\alpha}_j(t)\rangle \mathrm{d}\tau\,. \tag{8}$$

The ordering of the perturbations is given by

$$\begin{aligned}\boldsymbol{\delta\tilde{\alpha}} &= (\delta\tilde{\alpha}_1,\, \delta\tilde{\alpha}_2,\, \delta\tilde{\alpha}_3,\, \ldots,\, \delta\tilde{\alpha}_{13},\, \delta\tilde{\alpha}_{14})\\ &= (\delta\tilde{\alpha},\, \delta\tilde{\alpha}^\dagger,\, \delta\tilde{b}_1,\, \delta\tilde{b}_1{}^\dagger,\, \delta\tilde{b}_2,\, \delta\tilde{b}_2{}^\dagger,\, \delta\tilde{c}_1,\, \delta\tilde{c}_1{}^\dagger,\\ &\quad \delta\tilde{c}_2,\, \delta\tilde{c}_2{}^\dagger,\, \delta\tilde{J}_{23}{}^-,\, \delta\tilde{J}_2,\, \delta\tilde{J}_3,\, \delta\tilde{J}_{23}{}^+)\,.\end{aligned} \tag{9}$$

The calculation of the spectral matrix from the master equation by use of the generalized P representation[9] is standard. A detailed description of the method can be found in Ref. 10. The spectral matrix is given by

$$\mathbf{S}(\omega) = (\mathbf{A}_0 - i\omega\mathbf{I})^{-1}\mathbf{D}_0(\mathbf{A}_0{}^T + i\omega\mathbf{I})^{-1}\,, \tag{10}$$

where

$$A_{\zeta,\xi;0} = -\frac{\partial}{\partial\tilde{\alpha}_\xi}\dot{\tilde{\alpha}}_\zeta\bigg|_{\tilde{\boldsymbol{\alpha}}=\tilde{\boldsymbol{\alpha}}_0}$$

[here $\dot{\tilde{\alpha}}_\zeta$ means the right-hand side of the corresponding equation of Eqs. (5)], and we calculate the nonzero terms of the linearized, symmetric diffusion matrix $D_{i,j;0}$ to be

$$\begin{aligned}
D_{13,13;0} &= -2\tilde{g}\tilde{J}_{23;0}\tilde{\alpha}_0 + \gamma_{23}\tilde{J}_{3;0} + \Gamma\tilde{J}_{1;0}\,,\\
D_{12,12;0} &= -2\tilde{g}\tilde{J}_{23;0}\tilde{\alpha}_0 + \gamma_{12}\tilde{J}_{2;0} + \gamma_{23}\tilde{J}_{3;0}\,,\\
D_{13,12;0} &= 2\tilde{g}\tilde{J}_{23;0}\tilde{\alpha}_0 - \gamma_{23}\tilde{J}_{3;0}\,,\\
D_{14,14;0} &= D_{11,11;0} = 2g\tilde{J}_{23;0}\tilde{\alpha}_0\,,\\
D_{11,12;0} &= D_{14,12;0} = \gamma_{12}\tilde{J}_{23;0}\,,\\
D_{11,14;0} &= (\gamma_{12} + 2\gamma_p)\tilde{J}_{3;0} + \Gamma\tilde{J}_{1;0}\,,\\
D_{3,3;0} &= \epsilon_1\tilde{c}_1\,, \qquad D_{4,4;0} = \epsilon_1\tilde{c}_{01}{}^\dagger\,,\\
D_{5,5;0} &= \epsilon_2\tilde{c}_{02}\,, \qquad D_{6,6;0} = \epsilon_2\tilde{c}_{02}{}^\dagger\,,
\end{aligned} \tag{11}$$

where the ordering of the matrix is given by Eq. (9). We define the intensity squeezing spectrum by

$$V(\omega) = \int_{-\infty}^{\infty} \exp(i\omega\tau)\frac{\langle\hat{n}(t+\tau),\, \hat{n}(t)\rangle}{\overline{n}}\mathrm{d}\tau\,, \tag{12}$$

where we have used the notation $\langle X, Y\rangle = \langle XY\rangle - \langle X\rangle\langle Y\rangle$. The normalization constant, $\overline{n}$, is equal to the average photon flux striking the detector(s). This ensures that the standard quantum noise level is 1 in each case. If

$$\begin{aligned}\hat{n}(t) &= \hat{a}_{\mathrm{out}}{}^\dagger(t)\hat{a}_{\mathrm{out}}(t)\,,\\ \overline{n} &= 2\kappa_{ai}|\tilde{\alpha}_0|^2\,,\end{aligned} \tag{13}$$

we will obtain the laser spectrum. If we let

$$\begin{aligned}\hat{n}(t) &= \hat{c}_{i\mathrm{out}}{}^\dagger(t)\hat{c}_{i\mathrm{out}}(t)\,,\\ \overline{n} &= 2\kappa_{oi}|\tilde{c}_{0i}|^2\,,\end{aligned} \tag{14}$$

we will obtain the individual second-harmonic noise spectra. If we let

$$\begin{aligned}\hat{n}(t) &= \hat{c}_{1\mathrm{out}}{}^\dagger(t)\hat{c}_{1\mathrm{out}}(t) - \hat{c}_{2\mathrm{out}}{}^\dagger(t)\hat{c}_{2\mathrm{out}}(t)\,,\\ \overline{n} &= 2\kappa_{o1}|\tilde{c}_{01}|^2 + 2\kappa_{o2}|\tilde{c}_{02}|^2\,,\end{aligned} \tag{15}$$

we obtain the differenced noise spectrum of the two second-harmonic generators. Note that there are two detection schemes shown in Fig. 2(a). The direct detection scheme is phase insensitive and corresponds to Eqs. (15). The homodyne detection system is phase sensitive. By placing a phase delay in the local-oscillator arm one can observe both quadratures.

Classical noise is canceled on both quadratures. Equations (15) correspond to observing only the amplitude quadrature and the homodyne detection system. We examine only the second-harmonic output here, though similar results can be obtained for squeezing of the fundamental. Using the equivalent of Eqs. (7) for the output beams, we expand Eq. (12) to second order in the quantum fluctuations. To this order of approximation, intensity and amplitude spectra are equivalent. Using the input/output formalism of Collett and Gardiner,[11] we are able to relate the fluctuations of the output fields to those of the internal fields and thus obtain the spectra in terms of the spectral matrix $\mathbf{S}(\omega)$. Using the ordering of Eq. (9), we obtain for the laser spectrum

$$V_{\mathrm{las}}(\omega) = 1 + 2\kappa_{ai}[S_{12}(\omega) + S_{21}(\omega) + S_{11}(\omega) + S_{22}(\omega)], \tag{16}$$

for the second-harmonic spectra

$$\begin{aligned}V_{\mathrm{shg1}}(\omega) &= 1 + 2\kappa_{c1}[S_{78}(\omega) + S_{87}(\omega) + S_{77}(\omega) + S_{88}(\omega)],\\ V_{\mathrm{shg2}}(\omega) &= 1 + 2\kappa_{c2}[S_{9,10}(\omega) + S_{10,9}(\omega) + S_{9,9}(\omega)\\ &\qquad + S_{10,10}(\omega)],\end{aligned} \tag{17}$$

and for the differenced second-harmonic spectrum

$$V_-(\omega) = 1 + ((4\kappa_{o2}{}^2|\tilde{c}_{02}|^2)[S_{9,10}(\omega) + S_{10,9}(\omega) + S_{9,9}(\omega) + S_{10,10}(\omega)] + (4\kappa_{o1}{}^2|\tilde{c}_{01}|^2)[S_{78}(\omega) + S_{87}(\omega) + S_{77}(\omega) + S_{88}(\omega)] - 4\kappa_{o1}\kappa_{o2}\{\tilde{c}_{01}{}^*\tilde{c}_{02}{}^*[S_{7,9}(\omega) + S_{9,7}(\omega)] + \tilde{c}_{01}\tilde{c}_{02}{}^*[S_{8,9}(\omega) + S_{9,8}(\omega)] + \tilde{c}_{01}{}^*\tilde{c}_{02}[S_{7,10}(\omega) + S_{10,7}(\omega)] + \tilde{c}_{01}\tilde{c}_{02}[S_{8,10}(\omega) + S_{10,8}(\omega)]\})/(2\kappa_{o2}|\tilde{c}_{02}|^2 + 2\kappa_{o1}|\tilde{c}_{01}|^2), \tag{18}$$

where κ_{ok} is the decay rate of the output port being observed of the kth SHG crystal. Hence one can generate spectra numerically by first calculating the spectral matrix, using Eqs. (10) and (11) and the ordering of Eq. (9), and then substituting the relevant matrix elements into Eq. (16), (17), or (18).

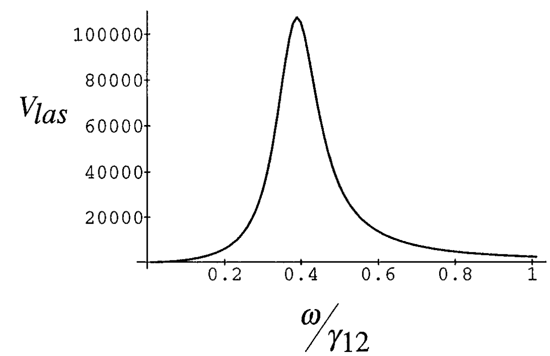
3. NOISE SPECTRA RESULTS

First we consider the perfect symmetry case in which $\kappa_{a1} = \kappa_{a2}$, $\kappa_{b1} = \kappa_{b2}$, $\kappa_{c1} = \kappa_{c2}$, and $\epsilon_1 = \epsilon_2$. The laser is given parameters typical of a solid-state laser such as Nd:YAG. The second-harmonic generators have good cavities at the fundamental but bad cavities at the second harmonic. At high enough pump powers self-pulsing of the second harmonic occurs.[4] At pump powers just below this critical point strong squeezing of the second harmonic is predicted if the pump is shot-noise limited. In Fig. 3 we examine the noise properties of our system near the critical point. Figure 3(a) shows the amplitude squeezing spectrum of the pump laser. A large relaxation oscillation 50 dB above the quantum noise level is present. Such large oscillations are typical in the spectra of solid-state lasers. Figure 3(b) shows the amplitude squeezing spectrum of the second-harmonic output of the first SHG cavity with and without detuning between the laser and the SHG fundamental. The laser noise appears in the SHG spectrum, obscuring the predicted squeezing. When a small detuning is introduced, additional noise is seen at low frequencies as a result of coupling between the laser phase noise and the SHG amplitude noise.[1] Figure 3(c) shows the differenced amplitude spectrum from the two SHG cavities. Noise that is due to both the laser intensity and the phase fluctuations is completely removed at all frequencies, while the squeezing is unaffected.

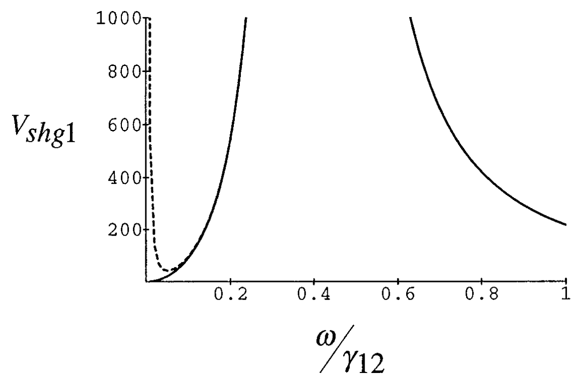
We now consider the effect of asymmetry between the two SHG cavities. First we consider the case in which one crystal receives more pump than the other. We model this by letting $\kappa_{a1} > \kappa_{a2}$. Hence SHG1 receives more pump and produces more second-harmonic photons. We balance detection by attenuating the output of SHG crystal 1 by reducing the decay rate of the output port (while keeping the overall cavity decay rate the same). This is equivalent to attenuating the beam after it leaves the cavity. After attenuation the individual noise spectra of the two second-harmonic generators are virtually identical. A small amount of classical noise now appears in the difference spectrum. The squeezing is also slightly reduced because of the attenuation of one of the beams [see Fig. 4(a)]. The effect is similar when the coupling constant (ϵ) is different for the two crystals.

Variations in the detuning between the second-harmonic generators have a more deleterious effect. Small changes in the detuning lead to large changes in the amount of laser phase noise that couples into the amplitude spectrum. This results in incomplete cancelation of the classical noise in the difference spectrum [see Fig. 4(b)], and the squeezing can no longer be retrieved at all frequencies.

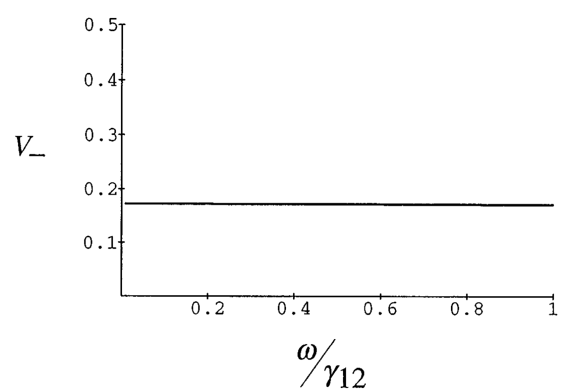
Another limitation to the effectiveness of this technique is the accuracy with which the detection system can be balanced. In Fig. 4(c) we show the effect on the direct detection scheme of one detector receiving 1%, 2%, and 5%



(a)



(b)



(c)

Fig. 3. Amplitude squeezing spectra for the system: (a) noise spectrum of the laser, (b) noise spectra of the second-harmonic output of SHG crystal 1. The solid curve has zero detuning, and the dashed curve has $\Delta\omega_1 = 0.1 \times \gamma_{12}$. (c) Difference noise spectrum of the second-harmonic generators. The parameters are $\kappa_{a1} = \kappa_{a2} = 10$, $\kappa_{b1} = \kappa_{b2} = 1$, $\kappa_{c1} = \kappa_{c2} = \kappa_{o1} = \kappa_{o2} = 10$, $\Delta\omega_1 = \Delta\omega_2 = 0.1$, $g_{23} = 300$, $\gamma_{23} = 0.0001$, $\gamma_p = 1000$, $\Gamma = 0.001$, and $\epsilon_1 = \epsilon_2 = 5000$, in units of γ_{12}.

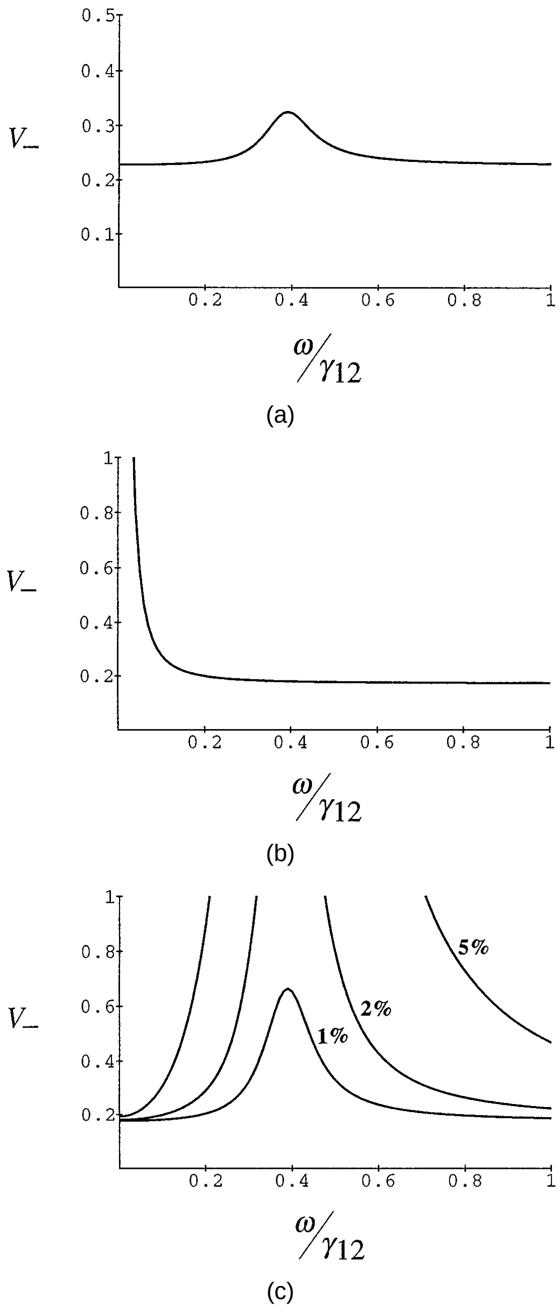


Fig. 4. Amplitude squeezing spectra for the system: (a) has an asymmetric pump ($\kappa_{a1} = 11$, $\kappa_{a2} = 9$, $\kappa_{b1} = \kappa_{b2} = 1$, $\kappa_{c1} = \kappa_{c2} = \kappa_{o1} = 10$, $\kappa_{o2} = 8.6$, $\Delta\omega_1 = \Delta\omega_2 = 0.1$, in units of γ_{12}); (b) has asymmetric detuning ($\kappa_{a1} = \kappa_{a2} = 10$, $\kappa_{b1} = \kappa_{b2} = 1$, $\kappa_{c1} = \kappa_{c2} = \kappa_{o1} = \kappa_{o2} = 10$, $\Delta\omega_1 = 0.09$, $\Delta\omega_2 = 0.11$); (c) has unbalanced detection [$\kappa_{a1} = \kappa_{a2} = 10$, $\kappa_{b1} = \kappa_{b2} = 1$, $\kappa_{c1} = \kappa_{c2} = \kappa_{o1} = 10$, $\Delta\omega_1 = \Delta\omega_2 = 0.1$, $\kappa_{o2} = 9.9$ (1%), $\kappa_{o2} = 9.8$ (2%), and $\kappa_{o2} = 9.5$ (5%), in units of γ_{12}]. Other parameters are the same as for Fig. 3.

less light than the other. We see that the system is quite sensitive to inaccuracies in the balancing of the detectors. Clearly the effect of all these asymmetries becomes more critical as the amount of classical noise present increases.

4. COUNTEREXAMPLE

With hindsight the basic result of Section 3 may seem obvious. However, care must be taken in choosing a system. As a counterexample we examine briefly an apparently similar setup for which squeezing cannot be retrieved. Consider the scheme of Fig. 5. The two SHG crystals still have separate cavities at the second harmonic but now have a common pump cavity at the fundamental. For simplicity the classically noisy pump is now modeled by a classical field coupled to a thermal reservoir. The master equation for this system is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = \frac{1}{i\hbar}[\hat{H}_{s1}, \hat{\rho}] + \frac{1}{i\hbar}[\hat{H}_{s2}, \hat{\rho}] \\ + \kappa_b(n_{\text{th}} + 1)(2\hat{b}\rho\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\rho - \rho\hat{b}^\dagger\hat{b}) \\ + \kappa_b n_{\text{th}}(2\hat{b}^\dagger\rho\hat{b} - \hat{b}\hat{b}^\dagger\rho - \rho\hat{b}\hat{b}^\dagger) \\ + \sum_{k=1,2}\kappa_{ck}(2\hat{c}_k\hat{\rho}\hat{c}_k{}^\dagger - \hat{c}_k{}^\dagger\hat{c}_k\hat{\rho} - \hat{\rho}\hat{c}_k{}^\dagger\hat{c}_k), \qquad (19)\end{aligned}$$

where now

$$\hat{H}_{sk} = i\hbar\,\frac{\epsilon_k}{2}\,(\hat{b}^{\dagger 2}\hat{c}_k - \hat{b}^2\hat{c}_k{}^\dagger)$$

and n_{th} is the number of thermal photons in the pump reservoir.

The difference spectrum of the two output beams can be calculated as before. The classical noise is still removed by difference detection, but now so is the quantum noise; so the difference trace simply gives the standard quantum limit (see Fig. 6). This is because the noise of the two SHG outputs are no longer independent, having been linked by the single fundamental cavity. The outputs from the two ends of a high-finesse cavity are correlated in the same way as the output ports of a beam splitter. As the conversion of the fundamental into the second harmonic is a coherent process, the correlation is transferred to the second harmonic; thus the output beams are not independent. In our original scheme [Fig. 2(a)] the correlation is destroyed by the independent incoherent decay processes of the two fundamental cavities.

It is interesting to note than an analogous setup to that of Fig. 5 that uses lasers pumped by a single cavity mode will produce independent beams and hence can be used to retrieve squeezing produced in the rate-matched regime.[10] This is because the conversion of pump photons into laser photons in a laser is not a coherent process.

5. DISCUSSION

We have shown rigorously that it is possible to produce optical beams with the correct properties such that optical or electronic subtraction will remove correlated classical noise to reveal underlying nonclassical behavior. In particular we have shown that it is possible to remove pump-induced intensity noise from the squeezing spectra of two SHG crystals by means of difference detection. The method removes noise introduced by both intensity and phase fluctuations of the pump laser. We find that the result is robust to pump asymmetries, provided that the detectors are accurately balanced. On the other hand, the system is quite sensitive to detuning asymmetries.

One way to avoid variations in the detuning would be to use a single SHG ring cavity. The pump laser output would be split at a 50/50 beam splitter and used a pump counterpropagating cavity modes. Provided that

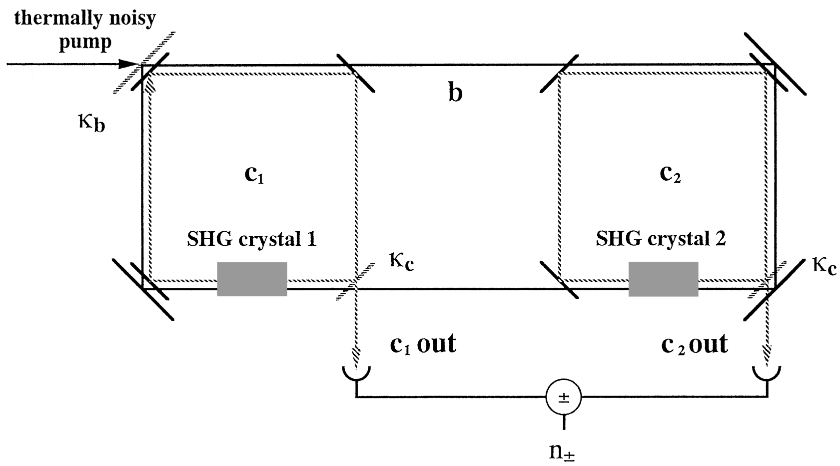


Fig. 5. Schematic representation of counterexample.

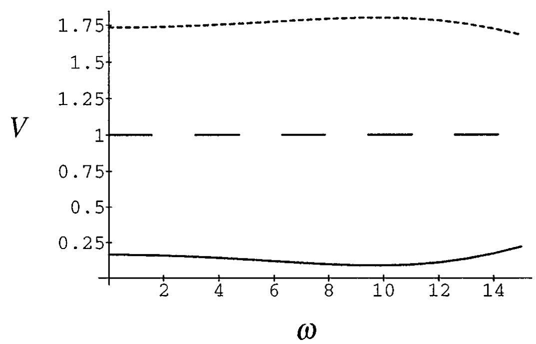


Fig. 6. Amplitude squeezing spectra for the counterexample system. The solid curve shows the spectrum of a single SHG cavity without thermal noise ($n_{\text{th}} = 0$), showing squeezing. The short-dashed curve shows the spectrum of a single SHG cavity with thermal noise ($n_{\text{th}} = 10$). Squeezing has been obscured. The long-dashed curve shows the difference spectrum of both cavities with or without thermal noise. Both the classical noise and the quantum noise have been removed, leaving the vacuum noise (shot noise). The parameters are $\epsilon_1 = \epsilon_2 = 5000$, $\kappa_b = 1$, $\kappa_c = 10$, and $E = 0.017$.

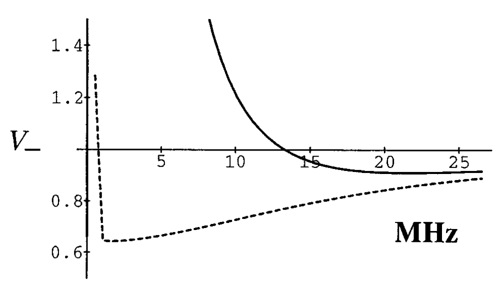


Fig. 7. Amplitude squeezing spectra for the system for conditions similar to those for Ref. 12. The solid curve is the output of one second-harmonic generator. The dashed curve is the differenced spectrum of both second-harmonic generators. The parameters are $\kappa_{a1} = 0.475$ and $\kappa_{a2} = 0.523$, where now we introduce an extra loss term in the laser (κ_{al}) such that the total loss rate of the laser is now $\kappa_{a1} + \kappa_{a2} + \kappa_{al} = 4.3$. Similarly, we have $\kappa_{b1} = \kappa_{b2} = 0.323$, where the total loss of the fundamental modes is now $\kappa_{bi} + \kappa_{bl} = 0.62$. Also, $\kappa_{c1} = \kappa_{c2} = 77.38$, $\kappa_{o1} = 50.3$, $\kappa_{o2} = 47.0$, $\Delta\omega_1 = \Delta\omega_2 = 0$, $g_{23} = 4.1 \times 10^5$, $\gamma_{23} = 0.00013$, $\gamma_p = 8.3 \times 10^6$, $\Gamma = 25 \times 10^{-8}$, and $\epsilon_1 = \epsilon_2 = 1.32 \times 10^5$, in units of γ_{12}.

we can ignore saturation effects and higher-order nonlinearities, this arrangement would be equivalent to that of Fig. 2(a). As both modes share the same mirrors, the detuning of the two modes will be identical. Also, because both modes are in the same crystal, any classical noise from the crystal itself will be correlated and may be removed by the difference detection.

Recent squeezing experiments using SHG's have been limited in their success, in part, because of the presence of pump noise.[12] In Fig. 7 we use parameters similar to those of Ref. 12 with the theory of Section 2. We include the quantum efficiencies of the detectors (65%). We assume a pump asymmetry of 10% and that the detection scheme is 2% off balance. With a single second-harmonic generator a squeezing maximum of 10% is observed at ~20 MHz. No squeezing is seen below 13 MHz. With difference detection of two SHG modes, squeezing of up to 40% is observed down to 1 MHz. The lack of perfect balance allows the relaxation noise of the laser to appear at frequencies below 1 MHz. This example shows that a significant increase in the magnitude-and-frequency range of the observed squeezing can be achieved in realistic situations.

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