Measurement of Quantum Weak Values of Photon Polarization

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We experimentally determine weak values for a single photon’s polarization, obtained via a weak measurement that employs a two-photon entangling operation, and postselection. The weak values cannot be explained by a semiclassical wave theory, due to the two-photon entanglement. We observe the variation in the size of the weak value with measurement strength, obtaining an average measurement of the $S_1$ Stokes parameter more than an order of magnitude outside of the operator’s spectrum for the smallest measurement strengths.

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It is commonly thought that the mean value of a quantum-mechanical measurement must be bounded by the extrema of a spectrum of eigenvalues, a consequence of statistical mathematics and the measurement postulate of quantum mechanics. However, there exist certain measurement outcomes for which this is not the case—these results are called weak values, since they arise as the outcomes of weak measurements on certain preselected and postselected quantum systems [1–11]. The canonical example of weak values is the gedanken experiment of Aharonov, Albert, and Vaidman [1], who described how it would be possible to use a weak measurement to measure (say) the $\sigma_z$ eigenvalue of a spin-1/2 particle, and determine an average value $\langle \sigma_z \rangle = 100$.

Weak values are an important and interesting phenomenon, because they assist us in understanding many counterintuitive results of quantum mechanics. For instance, weak values form a language by which we can resolve certain paradoxes and model strange quantum behavior. Important examples include: Hardy’s paradox [12,13], in which two particles that always annihilate upon meeting are sometimes paradoxically measured after this annihilation event; the apparent superluminal transport of pulses in optical fibres displaying polarization mode dispersion [9]; apparently superluminal particles travelling in vacuum [8]; and quantifying momentum transfer in twin-slit “which-path” experiments [14–16]. Weak values are useful in simplifying calculations wherever a system is weakly coupled to a monitored environment [7,9]. They also are an example of a manifestly quantum phenomenon, in that the analysis of weak values can lead to negative (pseudo-) probabilities [12], an effect never observed in analogous classical measurements.

Here we present the first unambiguously quantum-mechanical experimental realization of weak values, where we use a nondeterministic entangling circuit to enable one single photon to make a weak measurement of the polarization of another, subject to certain preselections and postselections. Previous demonstrations of weak values using electromagnetic radiation [17–20] have used coherent states and weak measurements arising from the coupling of two degrees of freedom of the photon. They can thus be explained semiclassically using a wave equation derived from Maxwell’s equations [21]. A cavity QED experiment [22] has been performed that was subsequently analyzed in terms of weak values [7], but the continuous spectrum precluded observations of anomalously large average values. By using two single photons, and realizing the weak measurement with a two-particle entangling operation, the weak values we measure (including extra-spectral weak values) are not able to be described in semiclassical terms—a crucial result in the experimental verification and study of the phenomenon.

The observable we measure is the polarization of a single photon in the horizontal-vertical ($H-V$) basis, i.e., the quantum operator corresponding to the $S_1$ Stokes parameter [23], $\hat{S}_1 = |H\rangle\langle H| - |V\rangle\langle V|$, with expectation value $\langle \hat{S}_1 \rangle = \langle \mu | \hat{S}_1 | \mu \rangle$ for some state $| \mu \rangle$. According to the standard quantum formulation of measurement [24], $-1 \leq \langle \hat{S}_1 \rangle \leq 1$ for any single-photon polarization state. We will find that it is possible, using weak measurements, to obtain average values for $\hat{S}_1$ far outside this range.

By analogy with the scheme of Aharonov, Albert, and Vaidman, we prepare the polarization of a single photon in the state

$$|\psi\rangle = |H\rangle \alpha + |V\rangle \beta,$$

where $|\alpha|^2 + |\beta|^2 = 1$. Subsequently, we make a weak, nondestructive measurement on the photon’s polarization in the $H - V$ basis. The weak measurement is made using a nondeterministic generalized photon polarization measurement device [25], which is deemed to have worked whenever a single photon is present at each of the signal and meter outputs. The generalized measurement device works by entangling the signal photon’s polarization with that of a meter photon prepared in the state $|\gamma|H\rangle + \gamma|V\rangle$, before measuring the meter photon’s polarization. Without loss of generality we choose $\gamma$ to be real; $\gamma^2 + \gamma^2 = 1$.

The experimental setup is shown in Fig. 1.

When operating with balanced modes [26], and after signal and meter photons interact but before either is...
Weakening measurement device, which operates via measurement made by interacting the photon with another (meter) photon in a classical interference at the $25,26]$. The entangling operation occurs because of the non-dentence window was 1 ns. We delivered the photons to the 1000 s when postselection on $j$.

It follows that, with measurement of the meter photon in the $H$ state and where, for example, $P(H|A)$ denotes the probability of measuring $H$ in the meter output given that postselection on signal state $|A\rangle$ was successful. The general expression for the expected postselected value is then

Using Eq. (2), it can be shown that if $\gamma \to 1/\sqrt{2}$, then $\lambda(\hat{S}_1) = \text{Re}((\alpha + \beta)/(\alpha - \beta))$ so that when $\alpha - \beta = 0$, the weak value of $\hat{S}_1$ can be arbitrarily large. In practice, it is necessary to operate with nonzero measurement strength and postselection probability, so that a precise experimental value for $\lambda(\hat{S}_1)$ can be obtained in a finite acquisition time. More detail on the theory of qubit weak values can be found in Ref. [27].

Postselected weak values are an important indicator of quantum behavior, since the bizarre results that can be obtained are not paralleled in the probabilities of analogous classical measurements. Large weak values arise from a quantum interference effect that results from the postselection of the signal photon state, which can be seen from the entangled state in Eq. (2). Consider the result when the meter photon is detected in the state $|H\rangle_m$, but no postselection is employed in the signal arm. The probability of this event is given by the expectation value of the projector
settings. However, since the calculated values of $A_j$, the error in strength, the uncertainty in nondemolition measurement followed by postselection on than would be expected for a strong quantum-

knowledge is identical to the and the remainder of the setup. The measurement strength from additional coincidence mea-

ments, to deal with errors in the input waveplate setting

practical restriction on the size of the data set [28].

We measured the weak value of the single-photon polarization for a range of measurement strengths, with a nominal input state $|\psi\rangle = \cos(42^\circ)|H\rangle + \sin(42^\circ)|V\rangle = 0.743|H\rangle + 0.669|V\rangle$. In principle, the experimental value of $\gamma$ can be determined from the meter input waveplate settings. However, since the calculated values of $A(\hat{S}_1)$ are very sensitive to $\gamma$, it is desirable to obtain the actual measurement strength from additional coincidence measurements, to deal with errors in the input waveplate setting and the remainder of the setup. The measurement strength is identical to the knowledge of the generalized measurement device, $K = P_{HH} + P_{VV} - P_{HV} - P_{VH} = 2\gamma^2 - 1$ (Ref. [25]), where, e.g., $P_{HV}$ is the probability of observing a horizontally and a vertically polarized photon at the signal and meter outputs of the device, respectively, and where these probabilities are measured with a signal input state $|D\rangle = (|H\rangle + |V\rangle)/\sqrt{2}$, and without postselecting the state $|A\rangle$. Because of Poissonian counting statistics in the measurement of $K$, the relative size of the error is quite large when $K$ is near zero.

The weak values for $\hat{S}_1$ were determined using Eq. (5) over a range of measurement strengths (Fig. 2). $P(H|A)$ and $P(V|A)$ were obtained from experimental coincidence measurements. For the smallest measurement strength, $K = 0.006$, we observed $A(\hat{S}_1) = 47$, which is much larger than would be expected for a strong quantum-nondemolition measurement followed by postselection on $|A\rangle$, i.e., $A(\hat{S}_1) = \alpha^*\alpha - \beta^*\beta = 0.1$, and also well outside the spectrum of $\hat{S}_1$.

The errors in $K = 2\gamma^2 - 1$ of approximately $\pm 0.015$ lead to substantial errors in the largest weak values, due to the form of Eq. (5). In fact, for the smallest measurement strengths, the uncertainty in $K$ encompasses $K = 0$, and the error in $\alpha(\hat{S}_1)$ is unbounded above. The triangles in Fig. 2 illustrate the “worst case” where each point is varied by $1\sigma$ in the value of $K$, in a direction that reduces the weak value. Even in this case, the smallest measurement strength yields a weak value of 19. In principle, the errors, which are all derived from Poissonian photon counting statistics, could be reduced arbitrarily by collecting larger samples of data. However, the low probability of the postselection, along with the very small correlation between the signal and meter photons, leads to very long collection times—a practical restriction on the size of the data set [28].

As the measurement strength is increased, we observe that the weak value of $S_1$ decreases until it is no longer greater than the strong value $|\alpha|^2 - |\beta|^2 \approx 0.1$. As noted in Ref. [25], the generalized measurement device does not exhibit perfect correlations between signal and meter due to imperfect mode matching. In the present case, this leads to a systematic offset in the weak value for large $K$, so that in fact it drops below this value.

The slight imperfections of the device mean that the theoretical weak value of Eq. (6), which assumes no mixture, does not completely describe the measurement. Instead, we determine the actual transfer matrix of the device—the process matrix—using quantum process tomography (in the manner of Ref. [29]). This provides an independent means of obtaining a parametric model of the postselected weak measurement process. As with Eq. (6), the expression obtained is parametrized by $\alpha$, $\beta$, $\gamma$, and $\tilde{\gamma}$, although the slight mixture leads to a lengthier form [30]. The calculated curve for $A(\hat{S}_1)$, plotted for our nominal input state, is shown in Fig. 2.

From a classical point of view, or even a typical quantum measurement point of view, it is quite strange that the measured expectation value of the $\hat{S}_1$ Stokes operator lies outside the interval $[-1, 1]$. The strangeness is perhaps more dramatic when we consider the results in terms of mean photon number. We can think of the expectation value of $\hat{S}_1$ as the difference in mode occupation between the two (spatially degenerate) $H$ and $V$ polarization modes. For instance, writing the strong measurement of $\hat{S}_1$ for a

FIG. 2. Variation of experimentally observed weak values of $S_1$ (circles) with measurement strength, $K = 2\gamma^2 - 1$. The error bars plotted arise from the effect of Poissonian counting statistics on $P(H|A)$ and $P(V|A)$. Bars not shown are smaller than the marker dimensions. In addition, errors in $K$ of approximately $\pm 0.015$ (over all values of $K$) lead to correlated errors in $\alpha(\hat{S}_1)$ via Eq. (5)—i.e., a displacement in $K$ due to error leads to a displacement in the weak value such that a given data point moves along a hyperbola. The triangles illustrate the worst case where each point is varied by $1\sigma$ in the value of $K$, in a direction that reduces the weak value. The dashed line indicates the predicted weak value, based on a model (with no free parameters) of the generalized measurement device obtained from quantum process tomography.
single photon in state $| \psi \rangle$ in terms of mode occupation number:

$$\langle \hat{S}_1 \rangle = \lambda \langle \hat{S} \rangle P(A) + P(\hat{S}) P(D) = \alpha \alpha^* - \beta \beta^*.$$  \hspace{1cm} (8)

Again, the effects of mixture in the experimental process mean that this relationship does not exactly hold for the data—for instance, the weak measurement device slightly decoheres the signal photon, resulting in $P(A) = 0.012$ instead of the expected value, which is $\frac{1}{2} - \alpha \beta^* = 0.0027$ when $2\gamma^2 - 1 = 0$. Using the expected value for $P(A)$, and the measured weak value $\lambda \langle \hat{S}_1 \rangle = 47$, we obtain $\langle \hat{S}_1 \rangle = 0.25$. The standard error for $\langle \hat{S}_1 \rangle$ is bounded below by 0.10 and is unbounded above due to the error in $\langle \hat{S}_1 \rangle$. Using the expression obtained from the tomographic reconstruction, we can determine an accurate expectation value for $\hat{S}_1$ from the measured weak value, even in the presence of mixture. We obtain $\langle \hat{S}_1 \rangle = 0.08 \pm 0.03$ by this method. From the input settings, we expect $\langle \hat{S}_1 \rangle = 0.10$.

In conclusion, we have demonstrated a completely quantum realization of weak values. The weak measurement step relies on nonclassical interference between signal and meter photons, meaning that the results cannot be explained by Maxwell’s equations alone. Using this technique, we observe expectation values of quantum observables far outside the range generally allowed by quantum measurement theory, including mean values of the single-photon $S_1$ Stokes parameter of up to 47.

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[26] Here we add a 2/3 loss to each of the noninteracting signal and meter modes, which simplifies the characterization by balancing all of the modes. The resulting small decrease in success probability is negligible compared with the low probability of later postselecting the signal in $|A\rangle$.
[28] With ultrabright quasi-phasematched waveguide sources [32] becoming available, this restriction may in the future be relaxed by several orders of magnitude.
[30] See EPAPS Document No. E-PRLTAO-94-018522 for the parametric expression obtained from tomographic reconstruction. A direct link to this document may be found in the online article’s HTML reference section. The document may also be reached via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information.
[31] Since our source is continuous-wave parametric downconversion, and based on the observed counting rates, the chance of two or more photons being present in the signal mode within the detection resolution time is negligibly small.